Single Pure - Coordinate Geometry Hard

1. Find the equation of the line that passes through $(-1, 2)$ and $(2, -3)$ in the form $ax+by+c = 0$.
$\boxed{5x + 3y - 1 = 0}$

2. Find the equation of the line that passes through $(p, 3p)$ and $(4p, -p)$ in the form $ax+by+c = 0$.
$\boxed{4x + 3y - 13p = 0}$

3. Find the equation of the line that lies perpendicular to $x - 3y + 4 = 0$ and passes through the point $(\frac{1}{2}, -1)$ in the form $ax + by + c = 0$.
$\boxed{6x + 2y - 1 = 0}$

4. Find the equation of the line that lies perpendicular to $x + ky + w = 0$ and passes through the point $(k, -1)$ in the form $ax + by + c = 0$.
$\boxed{kx - y - k^2 - 1 = 0}$

5. Find the intersection of the lines $3x - 2y = 7$ and $x + 3y = 1$.
$\boxed{(\frac{23}{11}, -\frac{4}{11})}$

6. Find the intersection of the lines $y = mx + c$ and $ax + by = 0$.
$\boxed{\left(-\frac{bc}{a+bm}, \frac{ac}{a+bm}\right)}$

7. Find the equation of the perpendicular bisector of the points $(2, 4)$ and $(4, 10)$ in the form $ax + by + c = 0$.
$\boxed{x + 3y - 24 = 0}$

8. Find the equation of the perpendicular bisector of the points $(\frac{1}{2}, -\frac{2}{3})$ and $(-1, \frac{1}{4})$ in the form $ax + by + c = 0$.
$\boxed{432x - 264y + 53 = 0}$

9. Find the equation of the perpendicular bisector of the points $(2q, 0)$ and $(2, -4)$ in the form $ax + by + c = 0$.
$\boxed{(q - 1)x + 2y + 5 + q^2 = 0}$

10. Find the point on the line $x + y = 4$ closest to the point $(3, 0)$.
$\boxed{(\frac{7}{2}, \frac{1}{2})}$

11. Find the point on the line $3x + y + 7 = 0$ closest to the point $(2, 0)$.
$\boxed{(-\frac{19}{10}, -\frac{13}{10})}$

12. Find the point on the line $x + 2y = 1$ closest to the point (a, b).
$\boxed{\left(\frac{1+4a-2b}{5}, \frac{2-2a+b}{5}\right)}$

13. Find the shortest distance from the point $(1, 1)$ to the line $y = 2x + 3$.
$\boxed{\frac{4\sqrt{5}}{5}}$

14. Find the shortest distance from the point $(4, 1)$ to the line $y = 2x$.
$\boxed{\frac{7\sqrt{5}}{5}}$

15. Find the shortest distance from the point $(a, 2)$ to the line $y = mx + 1$.
$\boxed{\frac{\sqrt{a^2m^4-2am^3+a^2m^2+m^2-2am+1}}{m^2+1}}$

16. Find the shortest distance between the parallel lines $y = 2x$ and $y = 2x + 10$.
$\boxed{2\sqrt{5}}$

17. Find the shortest distance between the parallel lines $y = mx$ and $y = mx + c$.
$\boxed{\frac{c\sqrt{m^2+1}}{m^2+1}}$

18. Find the point which is equidistant from $(0, 0)$, $(2, 2)$ and $(3, -2)$. [Hint: Think about the perpendicular bisectors.]
$\boxed{(\frac{21}{10}, -\frac{1}{10})}$

19. A triangle is bounded by the three lines
$$x + y + 1 = 0 \qquad y = 2x - 1 \qquad y = k$$
where k is a positive integer. For what values of k is the area of the triangle less than 2008? [Hamilton]
$\boxed{1 \leqslant k \leqslant 50}$

20. A quadrilateral is enclosed within the four lines
$$2y = x + 4 \qquad y = 2x - 4 \qquad 2y = x - 2 \qquad y = 2x + 2.$$
Find its area. [Maclaurin]
$\boxed{12}$